Optimal footfall patterns for cost minimization in running

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ABSTRACT

Optimality in footfall pattern use is often studied in relation to running performance and injury risk. The typical variables assessed (metabolic cost, impact force) represent only two of many potential variables runners might want to minimize situationally. Here we used optimal control theory to predict optimal model-based running mechanics with 44 different cost functions. We tallied which different footfall patterns, then examined which patterns minimized which types of cost functions. When the model wore shoes, rearfoot striking (RFS) was predicted by 57% of the cost functions and was consistently optimal for functions related to whole-body energy expenditure and peak joint contact forces. No other footfall pattern was predicted by more than 25% of the functions. Non-RFS patterns tended to be optimal for functions that gave equal weight to all muscles, avoiding localized muscle fatigue. Non-RFS patterns were also predicted when minimizing average joint contact forces. Similar predictions were seen when the model ran barefoot, where RFS was optimal for 55% of the functions. The results suggest that RFS is the most versatile footfall pattern (optimal for the greatest number of goals), and may explain why RFS is the most common pattern in recreational shoe runners. We argue that natural non-RFS runners are not necessarily behaving “sub-optimally”, but rather may be optimizing their gaits on factors not tested here (e.g., comfort, which is difficult to quantify). In addition, switching from RFS to non-RFS may reduce the joint load accumulated during a run if speed and step length are maintained.

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1. Introduction

The notion of optimality in human running has recently been studied in relation to the use of different footfall patterns and, relatedly, to running barefoot or in different types of shoes (Nigg, 2009; Altman and Davis, 2012; Lieberman, 2012). It has long been known that runners can and do use different footfall patterns. For example, the Muybridge motion-picture sequences circa 1880 clearly depict men and women running barefoot with a variety of footfalls (Muybridge, 1955). However, the mechanical and energetic consequences of using particular footfall patterns have only recently received substantial scientific attention.

The science of footfall patterns often focuses implicitly or explicitly on which pattern is “best” by metrics such as energy expenditure (e.g. Perl et al., 2012; Gruber et al., 2013b) and variables presumably related to injury risk such as ground reaction forces (GRF) (e.g. Freychat et al., 1996; Boyer et al., 2014). This approach is useful for examining the sensitivity of relevant running-related variables to footfall pattern, but is limited by the need to select a specific definition for “best” ahead of time, e.g. lowest metabolic rate or lowest GRF impact peak. Running can be quantitatively summarized by a wide variety of metrics related to performance (e.g. metabolic rate, heat dissipation, comfort, speed) and mechanical loading (e.g. GRF, muscle forces, joint loading). A runner may prioritize different metrics in different situations and it is currently unclear which footfall patterns are optimal for achieving these various goals.

In this study, we took a different approach to comparing footfall patterns and asked which pattern is the best most of the time, for the greatest number of potential goals a runner may want to achieve optimally. This question is well suited to the use of optimal control theory and predictive direct collocation simulations (Van den Bogert et al., 2011), which were our primary methods. Because most runners land on their heels (i.e. rearfoot striking, RFS) when running in shoes (Hasegawa et al., 2007; Larson et al., 2011; Kasper et al., 2013; De Almeida et al., 2014), we assumed that most runners are behaving optimally, and hypothesized that RFS would be optimal for > 50% of the tested goals when running in shoes. Because most runners switch to a non-RFS pattern when running barefoot on hard surfaces (Herzog, 1978; Hamill et al., 2011; Gruber et al., 2013a), we hypothesized that a non-RFS pattern would be optimal for > 50% of the goals when running barefoot.
2. Methods

2.1. Footfall pattern definitions

The footfall pattern can be defined in several ways. The simplest definition classifies patterns by which part of the foot touches the ground first: the heel (indicating RFS) or the metatarsal heads (indicating forefoot strike, FFS). Some authors have also defined midfoot striking (MFS), where the heel and metatarsal heads touch down at about the same time, and toe striking (TS), where the heel never contacts the ground (Lieberman, 2012). We adopted these definitions here, with MFS defined as the heel and metatarsal heads both touching down within 2% of the gait cycle. This threshold is arbitrary, but did not affect the distribution of footfall results within a 0.5–4.0% range.

2.2. Musculoskeletal model

We simulated running gaits using a sagittal plane musculoskeletal model (Fig. 1) with 10 rigid segments (pelvis, trunk, thighs, shanks, feet, toes), 12 degrees of freedom (DoF), and young adult female inertial properties (De Leva, 1996). The joints were actuated by 12 bilateral pairs of muscles. Each muscle was Hill-based (Hill, 1938), with a contractile component (CC) in series with an elastic component (SEC). Muscles produced forces in response to input excitation signals u according to the activation dynamics (He et al., 1991)

\[ a = \frac{(u/(1 + \tau_1(1 - u/(\tau_2)))/u - a) \tau}{1 + \tau_1(1 - u/(\tau_2))} \]

and the contractile dynamics

\[ F_U = f_U(L_{CC})(1 - L_{CC}) = F_U L_{SEC} + D_{SEC} L_{SEC} \]

2.5. Problem statement

Simulations were cast as optimal control problems to find the time-varying muscle excitations u and the associated model states x that minimized a scalar cost function J when operating on the model for time interval \([0, T]\):

\[ J = \int_0^T L(x, u, t) dt \]

The function L is the cost rate, specified by the user. The muscle excitations were bounded:

\[ 0 \leq u \leq 1 \]

and the system dynamics were enforced as a path constraint:

\[ f_x, x, u = 0 \]

2.6. Muscle model parameters

where \( a \) is the activation level, \( \tau_1 = 0.080-0.050/FT \) and \( \tau_1 = 0.095-0.060/FT \) are time constants for activation and deactivation, \( FT \) is the proportion of fast-twitch muscle fibers, \( F_0 \) is the maximum isometric force, \( L_{CC} \) and \( L_{SEC} \) are the CC and SEC lengths, and \( B_{SEC} \) is the SEC damping rate (Haeufler et al., 2014).

The functions \( f_U \) and \( f_E \) were the non-dimensional CC force-length, CC force-velocity, and SEC force-extension relationships, defined according to Chadwick et al. (2014), Hatze (1977), and Caldwell (1995), respectively.

Muscle-specific parameter values tuned to represent a young adult female (Anderson et al., 2007) are shown in Table 1. Muscle energy expenditure was calculated from activation and CC velocity according to Minetti and Alexander (1997).

### Table 1

Muscle model parameters. \( F_0 \) = maximum isometric force; \( FT \) = proportion of fast-twitch fibers; \( L_x \) = optimal contractile component length; \( L_u \) = unloaded series elastic component length; \( W \) = force-length parabola width.

<table>
<thead>
<tr>
<th>Muscle</th>
<th>Dynamics parameters</th>
<th>Moment arms (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( F_0 ) (N)</td>
<td>( FT ) (dimensionless)</td>
</tr>
<tr>
<td>Erector Spinae</td>
<td>2588</td>
<td>0.44</td>
</tr>
<tr>
<td>Rectus Abdominis</td>
<td>1500</td>
<td>0.60</td>
</tr>
<tr>
<td>Iliopsoas</td>
<td>1653</td>
<td>0.56</td>
</tr>
<tr>
<td>Gastrocnemius</td>
<td>4658</td>
<td>0.52</td>
</tr>
<tr>
<td>Rectus Femoris</td>
<td>849</td>
<td>0.68</td>
</tr>
<tr>
<td>Hamstrings</td>
<td>2486</td>
<td>0.36</td>
</tr>
<tr>
<td>Vasti</td>
<td>4723</td>
<td>0.59</td>
</tr>
<tr>
<td>Gastrocnemius</td>
<td>1914</td>
<td>0.56</td>
</tr>
<tr>
<td>Tibialis Anterior</td>
<td>674</td>
<td>0.30</td>
</tr>
<tr>
<td>Soleus</td>
<td>3586</td>
<td>0.14</td>
</tr>
<tr>
<td>Extensor Digitorum</td>
<td>510</td>
<td>0.30</td>
</tr>
<tr>
<td>Flexor Digitorum</td>
<td>711</td>
<td>0.20</td>
</tr>
</tbody>
</table>
symmetry, average running speed $v$, and step duration $T$:

$$x(T) = x(0) + vT$$  \hspace{1cm} (8)

$$u(T) = u(0)$$  \hspace{1cm} (9)

where $x(0)$ and $u(0)$ are mirrored copies of the initial states and controls at $t=0$, and $\hat{x}$ is a unit vector in the direction of horizontal pelvis translation.

2.6. Simulation method

The optimal control problem described in Section 2.5 was solved numerically using an implicit direct collocation method (Van den Bogert et al., 2011). The states and controls were discretized on a grid of $N$ nodes spaced evenly over the step duration. The system dynamics were discretized as a set of equality constraints on this grid using the backward Euler method, which has good convergence properties but a numerical damping artifact (final results were re-optimized using the midpoint Euler method, which lacks this artifact). The IPOPT algorithm (Wächter and Biegler, 2006) was used to find the nodal values of $x$ and $u$ that locally minimized the cost function. All simulations were performed with prescribed values of $v=3.17$ m/s and $T=0.36$ s, which were mean experimental measurements from humans running “normally and comfortably” barefoot over force platforms, with no instructions on footfall pattern (Miller et al., 2014). The breakdown of subjects who used RFS/MFS/FFS was $3/5/5$.

Simulations were performed using a variety of different cost functions to observe which costs were minimized by which footfall patterns. The first simulation was a data-tracking simulation, performed to verify that the model could run in a realistic fashion, and to serve as a starting point for the other simulations. The cost function for the data-tracking simulation was

$$J = \frac{1}{10^6} \sum_{j=1}^{10} \left( \frac{\mu_j - \mu_0}{\sigma_0} \right)^2 + \frac{w}{240} \sum_{j=1}^{24} u_j^{10}$$  \hspace{1cm} (10)

where $\mu_j$ and $\sigma_j$ are the mean and standard deviation (SD) for variable $i$ at time step $j$ from the same experimental data as above (Miller et al., 2014), and $\mu_0$ is the analogous variable in the simulation. The 10 variables included in the tracking term were the bilateral hip, knee, and ankle angles and horizontal and vertical GRF. The second term on the right-hand side of Eq. (10) approximates the maximum muscle excitation, which has been suggested as a general control parameter for human gait (Ackermann and van den Bogert, 2010). The data-tracking simulation was first performed on a coarse grid ($N=11$) with low muscle excitation weight ($w=0.1$). This simulation was run to convergence in IPOPT, then interpolated onto incrementally finer grids until reaching a final grid of $N=101$. The value of $w$ was then increased incrementally until the model’s gross metabolic cost was realistic for human running at $3.17$ m/s (under $4.5$ J/m/kg; American College of Sports Medicine, 2014).

The data-tracking result was used as the initial guess for predictive simulations that included tracking term in the cost function. Instead, the cost function was set to one of the 44 different criteria in four different general categories:

- Costs related to whole-body energy expenditure or work;
- Costs related to muscle excitation, activation, or force;
- Costs related to joint contact forces;
- Other selected costs relevant to effort level and/or mechanical loading.

The specific cost functions used are omitted here for brevity but are included in the Electronic Supplementary Material. All simulations were performed in Matlab (R2012b, MathWorks, Natick, MA, USA) running IPOPT 3.10.3 on a 2.9-GHz Intel Core i5 CPU with 32 GB of memory (iMac, Apple, Cupertino, CA, USA). Each simulation took 20 min on average.

2.7. Barefoot simulations

To simulate barefoot running, all 44 predictive simulations were repeated with the shoe mass ($0.3$ kg) subtracted from each foot, and ground contact model parameters adjusted ($A=3.0 \times 10^7$ N/m^2, $B=10$ s/m) to represent barefoot heel impact test data (Aerts and De Clercq, 1993).

3. Results

3.1. Data-tracking simulation

The data-tracking simulation deviated from the mean experimental joint angles and GRF by $0.85$ SD on average (Fig. 2). The model used an RFS pattern with a small vertical GRF impact peak ($1.75$ times bodyweight). The timing of muscle activity was generally consistent with normative on/off electromyography timing (Fig. 3; Novacheck, 1998; Cappellini et al., 2006). The gross metabolic cost was $4.36$ J/m/kg.

3.2. Predictive simulations: shod and barefoot running

In the 44 predictive simulations, whether barefoot or shod, no consistent trends were seen in which cost functions predicted MFS vs. FFS, so these results were combined into a “non-RFS” category. RFS was observed $25/44$ times ($57\%$) in the predictive shod simulations and $24/44$ times ($55\%$) in the predictive barefoot simulations. TS was never seen in any of the simulations. However, the simulation that maximized average horizontal speed was close to TS ($1\%$ of the step duration, both shod and barefoot).

The footfall patterns for all predictive simulations are summarized in Table 2. Cost functions consisting of summed muscle-related variables weighted according to muscle size and strength (e.g. metabolic cost, total joint contact force impulse) predicted an RFS, both shod and barefoot. Cost functions that gave equal weight to all muscles regardless of their size and strength (e.g. sum of muscle activations or muscle stresses) used a mix of footfalls but overall featured more non-RFS patterns than RFS patterns.

Minimizing peak joint contact force at the knee predicted an RFS for both shod and barefoot. However, minimizing the average knee joint contact force predicted an FFS for both shod and barefoot. The only simulation that switched from RFS when shod to non-RFS when barefoot minimized the peak hip joint contact force ($7.31$ bodyweights when shod with RFS, $7.37$ bodyweights when barefoot with FFS). Minimizing the metabolic cost predicted an RFS for both shod ($3.55$ J/m/kg) and barefoot ($3.51$ J/m/kg). The simulations that minimized the vertical GRF impact peak and loading rate were very similar kinematically (RMSE $3.5$ degrees) and both predicted FFS.

3.3. Additional “subjects”

A major limitation of computer simulation studies of human movement is that, with few exceptions (e.g. Neptune et al., 2000), they have typically been performed using a single generic model (i.e. one representative “subject”), which makes it difficult to gauge their statistical validity. To partially address this issue, the entire shod and barefoot simulation procedure was repeated using three additional models, with body segment anthropometric and muscle parameters representing a young adult male, older adult male, and older adult female (Anderson et al., 2007; Miller et al., 2014). Similar results were produced with all four models (Fig. 4): RFS was more common than any other footfall pattern (range $55–61\%$), whether shod or barefoot, but running barefoot consistently reduced the number of cost functions for which RFS was optimal (range $50–55\%$). TS was still never seen, and no consistent trends were seen concerning cost functions predicted RFS when shod but non-RFS when barefoot.

4. Discussion

It has long been known that humans run with different footfall patterns and that running barefoot vs. shod often changes the pattern used. Much of the recent research on footfalls and footwear in running has focused on the merits and drawbacks of using different patterns. In this study, we addressed this topic from the perspective of optimal control theory, and found that for 44 different cost functions motivated by maximizing performance or minimizing injury risk, RFS was optimal $57\%$ of the time when running in shoes. The merging of all non-RFS patterns into a single category (Fig. 4) perhaps underemphasizes how frequent RFS was...
seen in these results: the RFS pattern was seen over twice as often as MFS or FFS, and this result was consistent for all four “subjects” (model parameter sets). MFS and FFS have some distinctions in biomechanics (e.g. FFS typically eliminates the GRF impact peak entirely while MFS may not) and performance (e.g. nearly all sprinters use FFS), and should perhaps be treated as distinct categories. The observation that a footfall pattern resembling TS was only optimal for acceleration (maximizing speed) is consistent with footfall patterns typically seen in the sprinting.

Functions that weighted muscles by their size and strength tended to predict RFS, while functions that weighted all muscles equally tended to predict MFS or FFS (Table 2). Muscle-weighted cost functions approximate net metabolic energy expenditure and encourage relatively low activations of larger, stronger muscles such as soleus, which is likely more active during the stance phase when a runner uses FFS vs. RFS (Rooney and Derrick, 2013). This result suggests that RFS is optimal for minimizing whole-body effort (e.g. metabolic cost) while non-RFS is optimal for minimizing the average

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**Fig. 2.** Pelvis angle, joint angles, and ground reaction forces for the data-tracking simulation (solid lines). The shaded areas are two standard deviations around the mean for 14 human subjects running in a normal/comfortable fashion (Miller et al., 2014). The stride begins and ends at initial foot-ground contact. Ground reaction forces are expressed in % bodyweight.

**Fig. 3.** Muscle model excitations (solid lines) and activations (dashed lines) from the data-tracking simulation. Solid bars at the top of most panels are normative muscle “on/off” timing during running from electromyography (Novacheck, 1998; Cappellini et al., 2006). The stride begins and ends at initial foot-ground contact. Abd. = abdominus; Fem. = femoris; Gastroc. = gastrocnemius; Ant. = anterior; Dig. = digitorum.
level of muscle activity. It is unknown which of these goals is more important for running performance, and this theory would need to be examined with human experiments. However, RFS was recently shown to have a lower oxygen consumption rate than FFS when natural RFS runners ran shod at 3.0–3.5 m/s (Gruber et al., 2013b).

Our first hypothesis, that RFS would be optimal most often when running in shoes, was supported. An unexpected finding was that our second hypothesis, that a non-RFS pattern would be optimal 55% of the time, and was still observed twice as frequently as any other pattern in all four subjects. Three possible explanations for this result are

1. Using non-RFS patterns is a mistake, and all runners should use RFS all the time to maximize their odds of being optimal. However, we think this explanation is highly unlikely and would be unwise to use as an instruction to runners. Some important metrics were minimized using non-RFS patterns. For example, the average knee joint contact force was minimized with FFS, and we have recently argued that average joint loads are more relevant for joint degeneration than peak loads (Miller et al., 2014).

2. We speculate that the preference for RFS vs. non-RFS patterns, especially when shod, is related to limb anatomy and muscular properties, and that parameter combinations for which non-RFS patterns would be predominantly optimal might not have been included here. Although we studied more “subjects” (four) than is typical in modeling studies, the biological variance present in the running population was not fully represented here. Anatomical features or muscular properties that distinguish between natural RFS and non-RFS runners would be a valuable contribution to the literature.

3. We did not include all potential cost functions, and may have inadvertently excluded some functions that would have predicted non-RFS patterns. However, the predictive simulations included 44 cost functions, compared to three in our previous work (Miller et al., 2012), and encompassed a wide range of relevant criteria for running performance and mechanical loading. If anything, the cost functions were slightly biased towards non-RFS by including two objectives (maximize speed and minimize vertical GRF impact peak) that were highly unlikely to be optimal with RFS.

One important cost in running that was not included here is comfort. The model feels no pain, yet running RFS barefoot over hard surfaces is usually uncomfortable. It is also not uncommon for RFS to be very uncomfortable, even when shod, in natural non-RFS runners. Comfort is recognized as an important factor in footwear selection (Mündermann et al., 2002) but there is no clear consensus on how to quantify comfort from gait measurements. Progress in measuring and quantifying comfort would be another valuable contribution to the literature. We suspect comfort is a major explanatory factor for many subject-specific aspects of running mechanics.

It could be argued that the frequency of RFS was inflated by including cost functions that may seem too similar to predict different footfall patterns (e.g. the “weighted muscle excitation” group, #8–11 in Table 2). However, functions #8–11 are quite different in mathematical programming (linear, quadratic, nonlinear/non-quadratic, and min/max approximation functions, respectively) and do not predict the same gait mechanics (Ackermann and van den Bogert, 2010). There was thus no reason to expect this result, and it was not seen in any other “group” of cost functions (#4–7, 12–15, 16–19, 20–23, and 24–27). We therefore
stand by our conclusion that RFS was the most frequently and broadly optimal footfall pattern.

The present results agree with recent studies indicating that changing footfall pattern changes the load distribution within the lower limb (Rooney and Derrick, 2013; Stearne et al., 2014). The general picture seems to be that switching from RFS to non-RFS may reduce knee and possibly overall lower limb loading, at the cost of increased ankle loading. Whether these changes affect the incidence of particular injuries remains to be verified.

As with any modeling study, there are limitations to consider when interpreting the present results in relation to real human running. The model lacks some features of human anatomy (e.g. arms, non-sagittal motion) that may have affected the results. We have performed most of these simulations using a more detailed 3D model with coarser controls (Miller, 2014) and achieved similar results. We would therefore expect a more detailed model to ultimately suggest similar conclusions.

Additionally, we simulated only one speed–stride length combination. The same combination was enforced on all simulations because these variables have large effects on running mechanics and energetics (e.g. Mercer et al., 2005; Gutmann et al., 2006). In fact, many of the purported effects of barefoot running can actually be attributed to stride length (Thompson et al., 2014). Allowing speed and stride length to vary would thus complicate comparisons between simulations and the inference of conclusions on footfall effects specifically. Nonetheless, it is unknown if these results would generalize to other combinations or to the free selection of speed and stride length. However, imposing a "barefoot" stride length on the shod simulations likely did not have a major influence on the results: switching between RFS/non-RFS only changes stride length by about 2% (Gruber et al., 2013b), which is within normal stride-to-stride fluctuations during sustained running (Jordan et al., 2007), and increasing stride length by up to 20% does not usually change the footfall pattern (Derrick et al., 2000).

Having highlighted some limitations, the benefits of this simulation approach (implicit direction collocation) are worth highlighting as well. This method is fairly new in biomechanics (Van den Bogert et al., 2011) but can potentially transform the role of computer simulations in human movement science. We were able to perform ~350 simulations of periodic gait in ~130 h, on a single CPU. Our previous work using the traditional parameter optimization approach to such simulations (Pandy et al., 1992), with a similar model, coarser controls, larger periodicity violations, and more efficient programming language (Fortran), required ~600 h distributed over eight CPUs (Miller et al., 2012). Much can be learned about human movements from simulations (e.g. Neptune et al., 2009), but the broader use of simulations has been persistently hindered by limitations such as the CPU time required and uncertainty in the generalizability of single−subject results. Collocation methods can potentially address both of these limitations (Kaplan and Heegaard, 2001; Ackermann and van den Bogert, 2010; Van den Bogert et al., 2011).

In conclusion, RFS accomplished the greatest number of goals optimally, appearing in 57% of the shod predictive simulations and 55% of the barefoot simulations. This result suggests that RFS, at least for the set of cost functions tested, was the most versatile footfall pattern, and may explain why the great majority of shod recreational runners use RFS. We suggest future work is needed on (i) reconciling subjective reports of comfort with gait lab measurements and (ii) explaining why some runners naturally prefer non-RFS patterns even when shod. Finally, we suggest that the study of footfall patterns should focus on their task-specific merits and limitations. It is unlikely that any one pattern is globally optimal for broad objectives such as performance or injury prevention, both for populations and individuals.

Conflict of interest statement

The authors affirm that we have no personal, professional, or financial conflicts of interest related to the present work.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jbiosc.2014.05.063.

References

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